

Laws of Exponents - Reference Chart

A-LOE 1

$$x^0 = 1$$

$$x^1 = x$$

Exponents 0 and 1

These laws were discussed in the video called "Exponents in Algebra". The first law tells you that anything raised to the 0th power is always 1. The second law tells you that anything raised to the 1st power is just itself.

$$x^{-n} = \frac{1}{x^n}$$

Negative Exponents

This important law helps us understand that negative exponents are essentially repeated division. The practical application is that it lets you re-write an expression with a negative exponent in inverse form as 1 over the exact same expression with a positive exponent.

$$(x^m)^n = x^{mn}$$

Taking a Power of a Power

This law shows that if you have an exponential expression that is raised to another power, you can simplify it by multiplying the two exponents together.

$$x^m x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

Multiplying or Dividing Exponential Expressions

These laws show how to simplify exponential expressions that have the same base. The base MUST be exactly the same for these to work. The first law says that if the expressions are being *multiplied*, you can simplify by *adding* the exponents. The second law says that if the expressions are being *divided*, you can simplify by *subtracting* the bottom exponent from the top..

$$(xy)^m = x^m y^m$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

Distributing Exponents

These laws show how you can distribute (or un-distribute if you reverse them) a common exponent to different bases. The first law shows how distributing an exponent works with variables (or expressions) that are being *multiplied*. The second law shows how distributing an exponent works with variables (or expressions) that are being *divided*.

Calculating Integer Exponents

A-LOE 2

Instructions: Use the first three laws you learned in the video to calculate or simplify these expressions. Leave any fraction answers in fraction form.

$$1 \quad 8^1 - 5^0 = 8 - 1 = 7$$

$$2 \quad (2x)^{-1} = \frac{1}{2x}$$

$$3 \quad 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$4 \quad 2 + 2^{-1} = 2 + \frac{1}{2} = 2\frac{1}{2}$$

$$5 \quad (x + 3)^0 = 1$$

$$6 \quad 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$7 \quad 5^0 + 2^0 - 9^0 = 1 + 1 - 1 = 1$$

$$8 \quad 4^{-x} = \frac{1}{4^x}$$

$$9 \quad (x)(x^{-1}) = \frac{x}{x} = 1$$

$$10 \quad x^0 - 1^2 = 1 - 1 = 0$$

$$11 \quad 4^0 + 4^{-1} = 1 + \frac{1}{4} = 1\frac{1}{4}$$

$$12 \quad \frac{1}{2^{-1}} = \frac{1}{\frac{1}{2}} = 2$$

$$13 \quad 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$14 \quad (x^2 + 1)^1 = x^2 + 1$$

$$15 \quad a^{-b} = \frac{1}{a^b}$$

$$16 \quad (x^2)(x^{-2})^1 = \frac{x^2}{x^2} = 1$$

Simplifying a Power of a Power

A-LOE 3

Instructions: Use the fourth law you learned in the video lesson to simplify these expressions. (You'll also need to use the first three laws in some of the problems.)

$$1 \quad (x^2)^4 = x^{2 \cdot 4} = x^8$$

$$2 \quad (x^2)^{-1} = x^{2(-1)} = x^{-2} = \frac{1}{x^2}$$

$$3 \quad (2^2)^2 = 2^{2 \cdot 2} = 2^4 = 16$$

$$4 \quad (a^4)^3 = a^{4 \cdot 3} = a^{12}$$

$$5 \quad (x^{-1})^3 = x^{-3} = \frac{1}{x^3}$$

$$6 \quad (4^2)^{-1} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$7 \quad (3^2)^2 = 3^{2 \cdot 2} = 3^4 = 81$$

$$8 \quad (x^{-2})^{-4} = x^{(-2)(-4)} = x^8$$

$$9 \quad (y^5)^3 = y^{5 \cdot 3} = y^{15}$$

$$10 \quad (8^{-2})^0 = 8^{(-2)(0)} = 8^0 = 1$$

$$11 \quad ((x^2)^3)^4 = (x^{2 \cdot 3})^4 = (x^6)^4 = x^{24}$$

$$12 \quad (7^{-2})^{-1} = 7^{(-2)(-1)} = 7^2 = 49$$

$$13 \quad \frac{1}{(x^{-1})^2} = \frac{1}{x^{-2}} = \frac{1}{\frac{1}{x^2}} = x^2$$

$$14 \quad (a^2)^5 = a^{2 \cdot 5} = a^{10}$$

$$15 \quad (x^2)^{\frac{1}{2}} = x^{(2)\frac{1}{2}} = x^1 = x$$

$$16 \quad ((x^{-1})^{-1})^{-1} = x^{(-1)(-1)(-1)} = x^{-1} = \frac{1}{x}$$

Multiplying and Dividing Expressions with Exponents

A-LOE 4

Instructions: Use the 5th and 6th laws you learned in the video lesson to simplify these expressions.

$$1 \quad (y^x)(y^{2x}) = y^{x+2x} = y^{3x}$$

$$2 \quad \frac{x^2}{x^3} = x^{2-3} = x^{-1} = \frac{1}{x}$$

$$3 \quad (3^2)(3^2) = 3^{2+2} = 3^4 = 81$$

$$4 \quad (7^{-5})(7^4) = 7^{-5+4} = 7^{-1} = \frac{1}{7}$$

$$5 \quad \frac{a^7}{a^3} = a^{7-3} = a^4$$

$$6 \quad x^9 x^2 = x^{9+2} = x^{11}$$

$$7 \quad \frac{5^y}{5^x} = 5^{y-x}$$

$$8 \quad \frac{x^{2a}}{x^{-a}} = x^{2a-(-a)} = x^{3a}$$

$$9 \quad (x^{-1})(x^{-5}) = x^{-1+(-5)} = x^{-6} = \frac{1}{x^6}$$

$$10 \quad \frac{(x+1)^3}{(x+1)^2} = (x+1)^{3-2} = x+1$$

$$11 \quad b^{-2} b^2 = b^{-2+2} = b^0 = 1$$

$$12 \quad \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{-2} = \left(\frac{1}{2}\right)^{4+(-2)} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$13 \quad a^5 a^3 = a^{5+3} = a^8$$

$$14 \quad \frac{y^1}{y^x} = y^{1-x}$$

$$15 \quad \frac{x^{-5}}{x^{-5}} = x^{-5-(-5)} = x^0 = 1$$

$$16 \quad (a^8)(a^{-7}) = a^{8+(-7)} = a^1 = a$$

Distributing and 'Un-Distributing' Exponents

A-LOE 5

Instructions: Use the last two laws learned in the video to distribute the exponent.

$$1 \quad (2x)^2 = 2^2 x^2 = 4x^2$$

$$2 \quad \left(\frac{a}{b}\right)^4 = \frac{a^4}{b^4}$$

$$3 \quad \left(\frac{x}{4}\right)^2 = \frac{x^2}{4^2} = \frac{x^2}{16}$$

$$4 \quad (ab)^3 = a^3 b^3$$

$$5 \quad (5y)^{-2} = 5^{-2} y^{-2} = \frac{1}{25y^2}$$

$$6 \quad \left(\frac{2x}{3}\right)^2 = \frac{2^2 x^2}{3^2} = \frac{4x^2}{9}$$

$$7 \quad (6ab)^2 = 6^2 a^2 b^2 = 36a^2 b^2$$

$$8 \quad \left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$$

Instructions: Use the last two laws learned in the video to 'un-distribute' the common exponent.

$$1 \quad x^4 y^4 = (xy)^4$$

$$2 \quad \frac{a^2}{5^2} = \left(\frac{a}{5}\right)^2$$

$$3 \quad x^2 y^2 z^2 = (xyz)^2$$

$$4 \quad \frac{x^2}{(x-1)^2} = \left(\frac{x}{x-1}\right)^2$$

$$5 \quad \frac{b^x}{a^x} = \left(\frac{b}{a}\right)^x$$

$$6 \quad x^{-2} y^{-2} = (xy)^{-2} = \frac{1}{(xy)^2} \text{ or } \left(\frac{1}{xy}\right)^2$$

$$7 \quad 8^n y^n = (8y)^n$$

$$8 \quad \frac{x^2}{3^2 y^2} = \left(\frac{x}{3y}\right)^2$$

Laws of Exponents in Combination - Set 1

A-LOE 6

Instructions: Simplify these expressions using all the laws you learned in the video lesson. Be sure to show your work!

$$\begin{aligned} 1 \quad (3x)^2 + \frac{x^5}{x^3} \\ 3^2 x^2 + x^{5-3} \\ 9x^2 + x^2 \\ 10x^2 \end{aligned}$$

$$\begin{aligned} 2 \quad \left(\frac{a}{2}\right)^2 - \frac{1}{a^0} \\ \frac{a^2}{2^2} - \frac{1}{1} \\ \frac{a^2}{4} - 1 \end{aligned}$$

$$\begin{aligned} 3 \quad \left(\frac{a}{b^2}\right)^4 \\ \frac{a^4}{(b^2)^4} \\ \left(\frac{a^4}{b^8}\right) \end{aligned}$$

$$\begin{aligned} 4 \quad (a^2)^3 (a^4)^{-2} \\ a^{2 \cdot 3} \cdot a^{4(-2)} \\ a^6 \cdot a^{-8} \\ a^{6+(-8)} \\ a^{-2} \text{ or } \left(\frac{1}{a^2}\right) \end{aligned}$$

$$\begin{aligned} 5 \quad \left(\frac{x^{10}}{x^7}\right)(x^4)^3 \\ x^{10-7} \cdot x^{4 \cdot 3} \\ x^3 \cdot x^{12} \\ x^{3+12} \\ x^{15} \end{aligned}$$

$$\begin{aligned} 6 \quad (2b)^3 - \frac{b^{10}}{b^7} \\ 2^3 b^3 - b^{10-7} \\ 8b^3 - b^3 \\ 7b^3 \end{aligned}$$

$$\begin{aligned} 7 \quad (3x^2)^0 + (a^{-5})^0 \\ 1 + 1 \\ 2 \end{aligned}$$

$$\begin{aligned} 8 \quad (x^n)(x^0)(x^{-n}) \\ x^n \cdot 1 \cdot \frac{1}{x^n} \\ \frac{x^n}{x^n} \\ 1 \end{aligned}$$

$$\begin{aligned} 9 \quad \left(\frac{4}{y^3}\right)^2 + 4(y^{-1})^6 \\ \frac{4^2}{(y^3)^2} + 4y^{-6} \\ \frac{16}{y^6} + \frac{4}{y^6} = \left(\frac{20}{y^6}\right) \end{aligned}$$

$$\begin{aligned} 10 \quad \frac{2}{(x^2)^3} - \frac{x^0}{x^6} \\ \frac{2}{x^6} - \frac{1}{x^6} \\ \left(\frac{1}{x^6}\right) \end{aligned}$$

$$\begin{aligned} 11 \quad b^{-2} - \frac{b^4}{b^6} \\ \frac{1}{b^2} - b^{4-6} \\ \frac{1}{b^2} - b^{-2} \\ \frac{1}{b^2} - \frac{1}{b^2} = 0 \end{aligned}$$

$$\begin{aligned} 12 \quad (a^{-1})^{-3} (b^{-3})^1 \\ a^{(-1)(-3)} \cdot b^{(-3)(1)} \\ a^3 \cdot b^{-3} \\ \left(\frac{a^3}{b^3}\right) \end{aligned}$$

Laws of Exponents in Combination - Set 2

A-LOE 7

Instructions: Simplify these expressions using all the laws you learned in the video lesson. Be sure to show your work!

1 $(4a)^2 - (a^3)(a^{-1})$

$$4^2 a^2 - a^{3+(-1)}$$

$$16a^2 - a^2$$

$$\boxed{15a^2}$$

2 $(-2x)^2 + \frac{x^9}{x^7}$

$$(-2)^2 x^2 + x^{9-7}$$

$$4x^2 + x^2$$

$$\boxed{5x^2}$$

3 $\left(\frac{x^3}{y^2}\right)^{-2}$

$$\frac{x^{3(-2)}}{y^{(-2)(-2)}}$$

$$\frac{x^{-6}}{y^4} = \boxed{\frac{1}{x^6 y^4}}$$

4 $\frac{x^4}{x^5} + \left(\frac{a}{a^3}\right)^2$

$$x^{4-5} + \frac{a^2}{a^6}$$

$$x^{-1} + a^{-4}$$

$$\boxed{\frac{1}{x} + \frac{1}{a^4}}$$

5 $(x^{-n})(x^0) + (x^n)^2$

$$\frac{1}{x^n} \cdot 1 + x^{2n}$$

$$\boxed{\frac{1}{x^n} + x^{2n}}$$

6 $\frac{b^8}{b^0} + \frac{b^{15}}{b^7} + (b^2)^4$

$$b^8 + b^{15-7} + b^{2 \cdot 4}$$

$$b^8 + b^8 + b^8$$

$$\boxed{3b^8}$$

7 $(3b^2 + 7x^{-5})^0$

$$\boxed{1}$$

8 $\frac{y^{15}}{y^3} + (2y^4)^3$

$$y^{15-3} + 2^3 y^{4 \cdot 3}$$

$$y^{12} + 8y^{12}$$

$$\boxed{9y^{12}}$$

9 $(x^3)^{-3}(x^{-2})^{-5}$

$$x^{(3)(-3)} \cdot x^{(-2)(-5)}$$

$$x^{-9} \cdot x^{10}$$

$$x^{-9+10}$$

$$\boxed{x}$$

10 $\left(\frac{5x^3}{2y^2}\right)^3$

$$\frac{5^3 x^{3 \cdot 3}}{2^3 y^{2 \cdot 3}}$$

$$\boxed{\frac{125x^9}{8y^6}}$$

11 $\frac{9}{(3x^{-1})^2} - \frac{x^0}{x^{-2}}$

$$\frac{9}{3^2 x^{-2}} - \frac{1}{x^{-2}}$$

$$x^2 - x^2$$

$$\boxed{0}$$

12 $(b^9)^{-1} \left(\frac{b^5}{b^8}\right)$

$$(b^{-9})(b^{5-8})$$

$$(b^{-9})(b^{-3})$$

$$b^{-9+(-3)}$$

$$b^{-12} \text{ or } \boxed{\frac{1}{b^{12}}}$$